

Links to KS3 and KS4 National Curriculum

Key Stage 3

Through the mathematics content, pupils should be taught to:

Develop Fluency

Statements taken from the National Curriculum in England 2014
consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers, and roots
select and use appropriate calculation strategies to solve increasingly complex problems
use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
substitute values in expressions, rearrange and simplify expressions, and solve equations
move freely between different numerical, algebraic, graphical, and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability, and statistics

Reason Mathematically

Statements taken from the National Curriculum in England 2014
extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
identify variables and express relations between variables algebraically and graphically
make and test conjectures about patterns and relationships; look for proofs or counter-examples
begin to reason deductively in geometry, number, and algebra, including using geometrical constructions
interpret when the structure of a numerical problem requires additive, multiplicative, or proportional reasoning
explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally

Solve Problems

Statements taken from the National Curriculum in England 2014
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develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems

develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
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begin to model situations mathematically and express the results using a range of formal mathematical representations

select appropriate concepts, methods, and techniques to apply to unfamiliar and non-routine problem

Number

Pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
understand and use place value for decimals, measures, and integers of any size	N3.1 N3.2 N3.6
order positive and negative integers, decimals, and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, , ≤, ≥	N2.1 N3.1 N4.3 N5.2
use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property	N1.6 N1.7 N1.8
use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative	N1.1 N1.2 N1.3 N1.4 N2.1 N2.2 N3.3 N3.4 N3.5 N3.6 N4.1 N4.2 N4.4 N4.5 N4.6 N11.1
use conventional notation for the priority of operations, including brackets, powers, roots, and reciprocals	N1.5
recognise and use relationships between operations including inverse operations	N1.2 (EXT) N1.3 (EXT) N1.4 (EXT) N3.6
use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations	N8.1
interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or zero	N9.1
work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)	N5.1
define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%	N5.1 N6.1 N6.2 N6.3 N6.4
interpret fractions and percentages as operators	N4.7 N6.2 N6.3
use standard units of mass, length, time, money, and other measures, including with decimal quantities	N11.2 N11.3
round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]	N7.1 N7.2 N7.3

use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$	N7.5 N7.6
use a calculator and other technologies to calculate results accurately and then interpret them appropriately	N11.1
appreciate the infinite nature of the sets of integers, real and rational numbers	

Algebra

Pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
use and interpret algebraic notation, including: ab in place of $a \times b$ $3y$ in place of $y + y + y$ and $3 \times y$ a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$ $\frac{a}{b}$ in place of $a \div b$ coefficients written as fractions rather than as decimals brackets	A3.1
substitute numerical values into formulae and expressions, including scientific formulae	A3.9
understand and use the concepts and vocabulary of expressions, equations, inequalities, terms, and factors	A3.1
simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms multiplying a single term over a bracket taking out common factors expanding products of two or more binomials	A3.2 A3.3 A3.4 A3.5 A3.6 A4.1
understand and use standard mathematical formulae; rearrange formulae to change the subject	A3.12
model situations or procedures by translating them into algebraic expressions or formulae and by using graphs	A3.1 R4.2
use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)	A3.8 A3.10 A3.11
work with coordinates in all four quadrants	A1.1 A1.2
recognise, sketch, and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane	A2.1 A2.2 A2.3 A2.4 A4.5
interpret mathematical relationships both algebraically and graphically	A3.1 R4.2
reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically, and algebraically	A2.4 A2.5
use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations	A2.5 A4.5 A4.6 A9.1
find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs	A2.5 A4.6 A6.1 A6.2 R4.1 R4.2 R4.3
generate terms of a sequence from either a term-to-term or a position-to-term rule	A10.1

recognise arithmetic sequences and find the n th term	A10.1
recognise geometric sequences and appreciate other sequences that arise	A10.3

Ratio, Proportion & Rates of Change

Pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
change freely between related standard units [for example time, length, area, volume/capacity, mass]	N11.2 N11.3 N11.4
use scale factors, scale diagrams and maps	G6.6
express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1	N4.1 N4.2
use ratio notation, including reduction to simplest form	R1.1 R1.2
divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio	R1.3 R1.4
understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction	R1.1
relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions	R1.1
solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics	N6.4 N6.5 N6.6
solve problems involving direct and inverse proportion, including graphical and algebraic representations	R2.1 R2.2
use compound units such as speed, unit pricing and density to solve problems	R3.1 R3.2

Geometry & Measures

Pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)	G 2.1 G 2.2 G 2.3 G 2.4 G 2.5 G 4.1 G 4.3
calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes	G 2.1 G 2.2 G 2.3 G 2.4 G 2.5 G 3.1 G 3.3
draw and measure line segments and angles in geometric figures, including interpreting scale drawings	G 5.1 G 5.2 G 6.1 G 6.6
derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line	G 7.1 G 7.2
describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric	G 1.1 G 1.2
use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles	G 1.1 G 8.1
derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies	G 1.1 G 3.1
identify properties of, and describe the results of, translations, rotations and reflections applied to given figures	G 1.2 G 8.1 G 9.1 G 9.2 G 9.3
identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids	G 7.3 G 8.1 G 8.2 G 9.4
apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles	G 6.2
understand and use the relationship between parallel lines and alternate and corresponding angles	G 6.3
derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons	G 6.4 G 6.5
apply angle facts, triangle congruence, similarity, and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs	G 1.1 G 6.4 G 6.5 G 8.1 G 8.2 G 10.1
use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles	G 10.1 G 10.2 G 10.3

use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones, and spheres to solve problems in 3-D	G 1.3 G 1.4
interpret mathematical relationships both algebraically and geometrically	

Probability

Pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale	
understand that the probabilities of all possible outcomes sum to 1	
enumerate sets and unions/intersections of sets systematically, using tables, grids, and Venn diagrams	
generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities	

Statistics

Pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
describe, interpret, and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)	S 1.3
construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data	
describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs	

Key Stage 4

Through the mathematics content, pupils should be taught to:

Develop Fluency

Statements taken from the National Curriculum in England 2014

consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots **{and fractional indices}**

select and use appropriate calculation strategies to solve increasingly complex problems, including exact calculations involving multiples of π **{and surds}**, use of standard form and application and interpretation of limits of accuracy

consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions, **{and expressions involving surds and algebraic fractions}**

extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations, and inequalities

move freely between different numerical, algebraic, graphical, and diagrammatic representations, including of linear, quadratic, reciprocal, **{exponential and trigonometric}** functions

use mathematical language and properties precisely

Reason Mathematically

Statements taken from the National Curriculum in England 2014

extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically

extend their ability to identify variables and express relations between variables algebraically and graphically

make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **{and proofs}**

reason deductively in geometry, number, and algebra, including using geometrical constructions

interpret when the structure of a numerical problem requires additive, multiplicative, or proportional reasoning

explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally

assess the validity of an argument and the accuracy of a given way of presenting information

Solve Problems

Statements taken from the National Curriculum in England 2014
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develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems

develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts

make and use connections between different parts of mathematics to solve problems

model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions

select appropriate concepts, methods, and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem
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Number

In addition to consolidating subject content from key stage 3, pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
apply systematic listing strategies, {including use of the product rule for counting}	
{estimate powers and roots of any given positive number}	N7.5 (EXT)
calculate with roots, and with integer {and fractional} indices	N8.2 N8.3
calculate exactly with fractions, {surds} and multiples of π ; {simplify surd expressions involving squares [for example $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$] and rationalise denominators}	N4.4 N4.5 N4.6 N10.1 N10.2
calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer	N9.2
{change recurring decimals into their corresponding fractions and vice versa}	N5.3
identify and work with fractions in ratio problems	R1.1 R1.4 (EXT)
apply and interpret limits of accuracy when rounding or truncating, {including upper and lower bounds}	N7.4 N7.6 N7.7

Algebra

In addition to consolidating subject content from key stage 3, pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
simplify and manipulate algebraic expressions (including those involving surds {and algebraic fractions}) by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; {factorising quadratic expressions of the form $ax^2 + bx + c$} simplifying expressions involving sums, products, and powers, including the laws of indices	A3.7 A3.12 A3.13 A4.2 A4.3 A5.1 A11.1 A11.2 A11.3
know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments {and proofs}	A3.1 A3.2 A3.3 A3.4 A3.5 A3.6 A3.12 A3.13 A4.1 A4.2 A5.1 A13.1
where appropriate, interpret simple expressions as functions with inputs and outputs; {interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}	A3.8 A12.1
use the form $y = mx + c$ to identify parallel {and perpendicular} lines; find the equation of the line through two given points, or through one point with a given gradient	A2.6 A2.7
identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically {and turning points by completing the square}	A4.4 A4.6 A4.7 A4.8 A4.9 A4.10 A4.11 A4.12 A4.13
recognise, sketch, and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, {the exponential function $y = k^x$ for positive values of k, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size}	A2.1 A2.2 A2.3 A2.4 A2.5 A2.6 A2.7 A4.5 A4.6 A4.7 A4.8 A4.9 A4.10 A4.11 A4.12 A4.13 A5.2 A6.1 A6.2 A6.4
{sketch translations and reflections of the graph of a given function}	A6.5
plot and interpret graphs (including reciprocal graphs {and exponential graphs}) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed, and acceleration	R4.3 R4.4 R4.5

{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs, and graphs in financial contexts}	R4.5
{recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point}	A6.3
solve quadratic equations {including those that require rearrangement} algebraically by factorising, {by completing the square and by using the quadratic formula}; find approximate solutions using a graph	A4.6 A4.7 A4.8 A4.9 A4.10 A4.11 A4.12 A11.1/2/3 (EXT)
solve two simultaneous equations in two variables (linear/linear {or linear/quadratic}) algebraically; find approximate solutions using a graph	A4.7 A9.1 A9.2
{find approximate solutions to equations numerically using iteration}	A8.1 A8.2
translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution	A3.1 A3.10 A3.11 A4.8 A4.9 A4.10 A4.11 A4.12
solve linear inequalities in one {or two} variable{s}, {and quadratic inequalities in one variable}; represent the solution set on a number line, {using set notation and on a graph}	A7.1 A7.2 A7.3 A7.4
recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a positive rational number {or a surd}) {and other sequences}	A10.2 A10.3
deduce expressions to calculate the n th term of linear {and quadratic} sequences	A10.1 A10.2

Ratio, Proportion & Rates of Change

In addition to consolidating subject content from key stage 3, pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)	
convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts	R3.1 R3.2 R3.3
understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; {construct and} interpret equations that describe direct and inverse proportion	R2.3 R2.4
interpret the gradient of a straight-line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion	R4.1 R4.2 R4.3 R4.4
{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic, and graphical contexts}	R4.5
set up, solve, and interpret the answers in growth and decay problems, including compound interest {and work with general iterative processes}	N6.7 A8.2

Geometry & Measures

In addition to consolidating subject content from key stage 3, pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
interpret and use fractional {and negative} scale factors for enlargements	G 9.4 G 9.5
{describe the changes and invariance achieved by combinations of rotations, reflections, and translations}	G 8.1 G 9.1 G 9.2 G 9.3
identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector, and segment	G 3.1 G 3.3
{apply and prove the standard circle theorems concerning angles, radii, tangents, and chords, and use them to prove related results}	G 12.1 G 12.2
construct and interpret plans and elevations of 3D shapes	G 1.4
interpret and use bearings	G 6.6
calculate arc lengths, angles, and areas of sectors of circles	G 3.2 G 3.4
calculate surface areas and volumes of spheres, pyramids, cones, and composite solids	G 4.1 G 4.2 G 4.3 G 4.4
apply the concepts of congruence and similarity, including the relationships between lengths, {areas and volumes} in similar figures	G 5.3 G 8.1 G 8.2
apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two {and three} dimensional figures	A1.3 G 10.1 G 10.2 G 10.3 G 10.4 G 10.5 G 10.6
know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° ; know the exact value of $\tan \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ$ and 60°	G 10.4
{know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles}	G 10.5 G 10.6
{know and apply $Area = \frac{1}{2} ab \sin C$ to calculate the area, sides, or angles of any triangle}	G 10.7
describe translations as 2D vectors	G 9.3 G 11.1
apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}	G 11.1

Probability

In addition to consolidating subject content from key stage 3, pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one	
use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size	
calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions	
{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}	

Statistics

In addition to consolidating subject content from key stage 3, pupils should be taught to:

<u>Subject Content taken from the National Curriculum in England 2014</u>	<u>Lessons Seen</u>
infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling	S 1.1 S 1.2 S 1.3
interpret and construct tables and line graphs for time series data	
{construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}	
interpret, analyse, and compare the distributions of data sets from univariate empirical distributions through: appropriate graphical representation involving discrete, continuous, and grouped data, {including box plots} appropriate measures of central tendency (including modal class) and spread {including quartiles and inter-quartile range}	
apply statistics to describe a population	S 1.2
use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing	